

DUBAI INTERNATIONAL ACADEMY

DREAM 🕢 INSPIRE 🔞 ACHIEVE

Re-Sit EXAMINATION-2014

DP1 IB DIPLOMA PROGRAMME

Yr 12 MATHEMATICS HL

Paper 2 – Calculator allowed

Time: 2 hours
Name:
INSTRUCTIONS TO CANDIDATES

Do Section A on the question paper and Section B on separate sheets

			Sec	tion A	1							Section B			_
4,	Q	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Total
	1								,						
Max	7	7	6	6	6	5	7	7	Ø	7	7	11	22	15	120
/lark_															
/lark															
Obta															
ined															







(a)	Find the first term and the common difference.
(b)	The sum of the first n terms exceeds 5000. Find the least possible value of n .

	•••••••••••••••••••••••••••••••••••••••
	•••••••••••••••••••••••••••••••••••••••
	•••••••••••••••••••••••••••••••••••••••
	•••••••••••••••••••••••••••••••••••••••
	•••••••••••••••••••••••••••••••••••••••



	der $f(x) = \ln x - e^{\cos x}$, $0 < x \le 10$.
a)	Sketch the graph of $y = f(x)$, stating the coordinates of any maximum and minimum points and points of intersection with the x-axis.
(b) S	olve the inequality $\ln x \le e^{\cos x}$, $0 < x \le 10$.
(b) S	olve the inequality $\ln x \le e^{\cos x}$, $0 < x \le 10$.
(b) S	olve the inequality $\ln x \le e^{\cos x}$, $0 < x \le 10$.
(b) S	



3 . [Maximum mark: 6]

(a)	Prove the trigonometric identity $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$.	[4]
-----	---	-----

(b) Given
$$f(x) = \sin\left(x + \frac{\pi}{6}\right) \sin\left(x - \frac{\pi}{6}\right)$$
, $x \in [0, \pi]$, find the range of f .

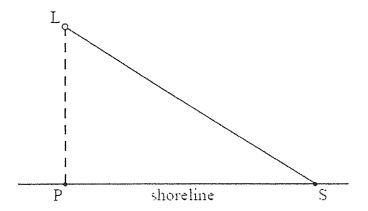
	••••••
1	



		**
[Ma:	ximum mark: 6]	
A co	mplex number z is given by $z = \frac{a+i}{a-i}$, $a \in \mathbb{R}$.	
(a)	Determine the set of values of a such that	
	(i) z is real;	
	(ii) z is purely imaginary.	[4]
(b)	Show that $ z $ is constant for all values of a .	[2]
•		
:		

|--|

5. A lighthouse L is located offshore, 500 metres from the nearest point P on a long straight shoreline. The narrow beam of light from the lighthouse rotates at a constant rate of 8π radians per minute, producing an illuminated spot S that moves along the shoreline. You may assume that the height of the lighthouse can be ignored and that the beam of light lies in the horizontal plane defined by sea level.



When S is 2000 metres from P,

(a) show that the speed of S, correct to three significant figures, is 214 000 metres per minute;

(5)

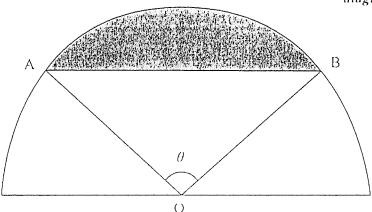
(b) find the acceleration of S.

(3) (Total 8 marks)

: b · [Maximum mark: 5]

The diagram below shows a semi-circle of diameter 20 cm, centre O and two points A and B such that $\triangle B = \theta$, where θ is in radians.

diagram not to scale

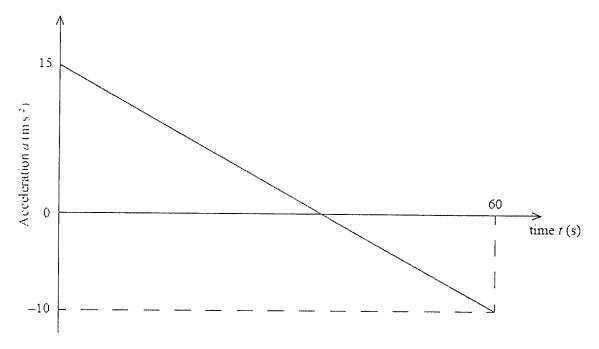


- (a) Show that the shaded area can be expressed as $50\theta 50\sin\theta$. [2]
- (b) Find the value of θ for which the shaded area is equal to half that of the unshaded area, giving your answer correct to four significant figures. [3]

 			 		 							 		 			 			 		 		 	 				 		 		 							 <u> </u>
					 																									. ,										
					 																											•			٠.					
					 												 										 •				 •									
			 ٠		 		٠	• •												 		 					 •				 ٠		 •			٠	•			
	•	•		•	 		٠	•		٠		 ٠	•		•	•	 	•	•	 •	•	 •		 •	 •	•	 ٠	•	 ٠		 ٠	•	 ٠	•		•			•	
	•	٠	 ٠	٠	 	•	•	•	•	•	•	 ٠	•	 •	•	•	 •	•	٠	 •	•	 •	•	 •	 •	•	 ٠	•	 ٠		•	•	 •	•		•	•		•	
																																						• •		
					 												 			 		 	•																	



7. A jet plane travels horizontally along a straight path for one minute, starting at time t = 0, where t is measured in seconds. The acceleration, a, measured in m s⁻², of the jet plane is given by the straight line graph below.



- (a) Find an expression for the acceleration of the jet plane during this time, in terms of t.
- (b) Given that when t = 0 the jet plane is travelling at 125 m s⁻¹, find its maximum velocity in m s⁻¹ during the minute that follows. (4)
- (c) Given that the jet plane breaks the sound barrier at 295 m s⁻¹, find out for how long the jet plane is travelling greater than this speed.

 (3)

 (Total 8 marks)

(1)

Use the method of mathematical induction to prove that $5^{2n} - 24n - 1$ is divisible by 576 for $n \in \mathbb{Z}^+$.
•••••••••••••••••••••••••••••••••••••••
•••••••••••••••••••••••••••••••••••••••
••••••
•••••

	(a)	The graph of $y = \ln(x)$ is transformed into the graph of $y = \ln(2x + 1)$. Describe two transformations that are required to do this.	
	(b)	Solve $ln(2x + 1) > 3 cos(x), x \in [0, 10].$	
			(Total d mar
	* •		
	- •	• • • • • • • • • • • • • • • • • • • •	
_			

(a) In how many different ways can the books be arranged?	
(b) In how many of these will the dictionary be next to the Mathematics books?	(Total 7 ma

.

11	·[Maximum	mark:	7]
----	-----------	-------	----

By using the substitution $x = 2 \tan u$, show that $\int \frac{dx}{x^2 \sqrt{x^2 + 4}} = \frac{-\sqrt{x^2 + 4}}{4x} + C$.

,	
•	
,	



Do NOT write solutions on this page.

SECTION B

Answer all questions on the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 11]

- (a) (i) Express the sum of the first n positive odd integers using sigma notation.
 - (ii) Show that the sum stated above is n^2 .
 - (iii) Deduce the value of the difference between the sum of the first 47 positive odd integers and the sum of the first 14 positive odd integers.

[4 marks]

- (b) A number of distinct points are marked on the circumference of a circle, forming a polygon. Diagonals are drawn by joining all pairs of non-adjacent points.
 - (i) Show on a diagram all diagonals if there are 5 points.
 - (ii) Show that the number of diagonals is $\frac{n(n-3)}{2}$ if there are *n* points, where n > 2.
 - (iii) Given that there are more than one million diagonals, determine the least number of points for which this is possible.

[7 marks]

Do NOT write solutions on this page.

13. [Maximum mark: 22]

A function f is defined by $f(x) = \frac{1}{2} (e^x + e^{-x}), x \in \mathbb{R}$.

- Explain why the inverse function f^{-1} does not exist. (a) (i)
 - Show that the equation of the normal to the curve at the point P where $x = \ln 3$ (ii) is given by $9x + 12y - 9 \ln 3 - 20 = 0$.
 - (iii) Find the x-coordinates of the points Q and R on the curve such that the tangents at Q and R pass through (0, 0). [14]
- The domain of f is now restricted to $x \ge 0$.
 - Find an expression for $f^{-1}(x)$. (i)
 - Find the volume generated when the region bounded by the curve y = f(x)and the lines x = 0 and y = 5 is rotated through an angle of 2π radians about the y-axis. [8]



14. The function f is defined by

$$f(x) = (x^3 + 6x^2 + 3x - 10)^{\frac{1}{2}}$$
, for $x \in D$,

where $D \subseteq \mathbb{R}$ is the greatest possible domain of f.

(a) Find the roots of
$$f(x) = 0$$
. (2)

(b) Hence specify the set
$$D$$
. (2)

(c) Find the coordinates of the local maximum on the graph
$$y = f(x)$$
. (2)

(d) Solve the equation
$$f(x) = 3$$
.

(e) Sketch the graph of
$$|y| = f(x)$$
, for $x \in D$. (3)

(f) Find the area of the region completely enclosed by the graph of
$$|y| = f(x)$$
.

(3)

(Total 15 marks)