



DUBAI INTERNATIONAL ACADEMY

DREAM INSPIRE ACHIEVE

Re-Sit EXAMINATION- 2014

DP1 IB DIPLOMA PROGRAMME

Yr 12 MATHEMATICS HL

Paper 2 – Calculator allowed

Time : 2 hours

Name: _____

INSTRUCTIONS TO CANDIDATES

Do Section A on the question paper and Section B on separate sheets

	Section A											Section B			Total
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Total
Max Mark	7	7	6	6	6	5	7	7	7	7	7	11	22	15	120
Mark Obtained															



2. [Maximum mark: 7]

Consider $f(x) = \ln x - e^{\cos x}$, $0 < x \leq 10$.

- (a) Sketch the graph of $y = f(x)$, stating the coordinates of any maximum and minimum points and points of intersection with the x -axis. [5]

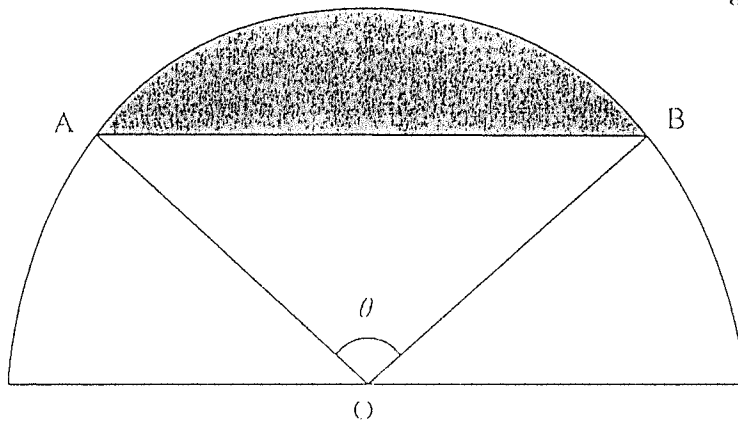
- (b) Solve the inequality $\ln x \leq e^{\cos x}$, $0 < x \leq 10$. [2]



6. [Maximum mark: 5]

The diagram below shows a semi-circle of diameter 20 cm, centre O and two points A and B such that $\angle AOB = \theta$, where θ is in radians.

diagram not to scale



- (a) Show that the shaded area can be expressed as $50\theta - 50\sin\theta$. [2]
- (b) Find the value of θ for which the shaded area is equal to half that of the unshaded area, giving your answer correct to four significant figures. [3]

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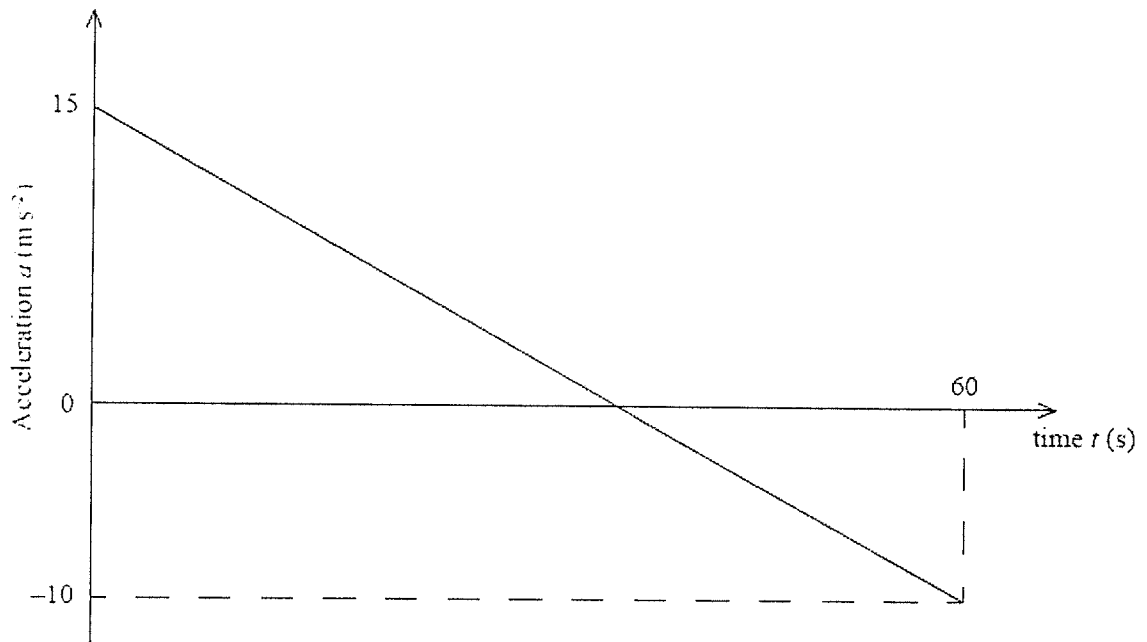
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Turn over

7. A jet plane travels horizontally along a straight path for one minute, starting at time $t = 0$, where t is measured in seconds. The acceleration, a , measured in m s^{-2} , of the jet plane is given by the straight line graph below.



- (a) Find an expression for the acceleration of the jet plane during this time, in terms of t . (1)
- (b) Given that when $t = 0$ the jet plane is travelling at 125 m s^{-1} , find its maximum velocity in m s^{-1} during the minute that follows. (4)
- (c) Given that the jet plane breaks the sound barrier at 295 m s^{-1} , find out for how long the jet plane is travelling greater than this speed. (3)
- (Total 8 marks)**

9. (a) The graph of $y = \ln(x)$ is transformed into the graph of $y = \ln(2x + 1)$. Describe two transformations that are required to do this.

(2)

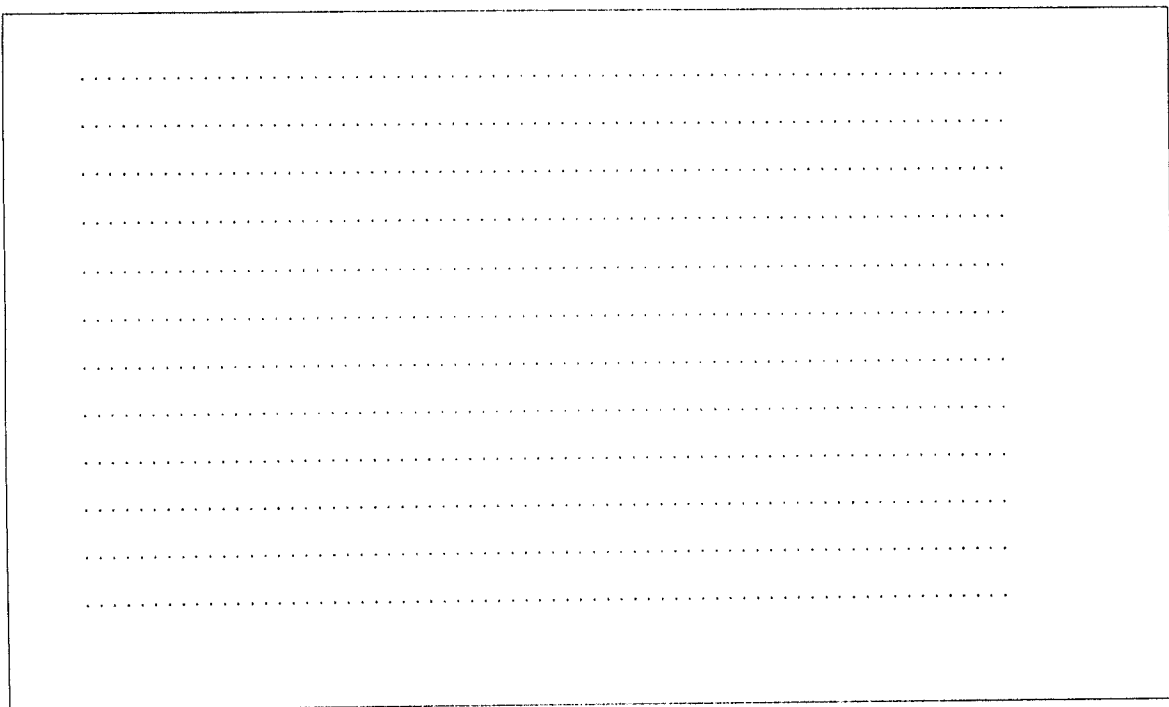
- (b) Solve $\ln(2x + 1) > 3 \cos(x)$, $x \in [0, 10]$.

(4)
(Total 6 marks)

10. Three Mathematics books, five English books, four Science books and a dictionary are to be placed on a student's shelf so that the books of each subject remain together.

(a) In how many different ways can the books be arranged? (4)

(b) In how many of these will the dictionary be next to the Mathematics books? (3)
(Total 7 marks)



Do **NOT** write solutions on this page.

SECTION B

Answer **all** questions on the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 11]

- (a) (i) Express the sum of the first n positive odd integers using sigma notation.
- (ii) Show that the sum stated above is n^2 .
- (iii) Deduce the value of the difference between the sum of the first 47 positive odd integers and the sum of the first 14 positive odd integers. [4 marks]
- (b) A number of distinct points are marked on the circumference of a circle, forming a polygon. Diagonals are drawn by joining all pairs of non-adjacent points.
- (i) Show on a diagram all diagonals if there are 5 points.
- (ii) Show that the number of diagonals is $\frac{n(n-3)}{2}$ if there are n points, where $n > 2$.
- (iii) Given that there are more than one million diagonals, determine the least number of points for which this is possible. [7 marks]



Do **NOT** write solutions on this page.

13. [Maximum mark: 22]

A function f is defined by $f(x) = \frac{1}{2}(e^x + e^{-x})$, $x \in \mathbb{R}$.

- (a) (i) Explain why the inverse function f^{-1} does not exist.
- (ii) Show that the equation of the normal to the curve at the point P where $x = \ln 3$ is given by $9x + 12y - 9 \ln 3 - 20 = 0$.
- (iii) Find the x -coordinates of the points Q and R on the curve such that the tangents at Q and R pass through $(0, 0)$. [14]

(b) The domain of f is now restricted to $x \geq 0$.

- (i) Find an expression for $f^{-1}(x)$.
- (ii) Find the volume generated when the region bounded by the curve $y = f(x)$ and the lines $x = 0$ and $y = 5$ is rotated through an angle of 2π radians about the y -axis. [8]



14. The function f is defined by

$$f(x) = (x^3 + 6x^2 + 3x - 10)^{\frac{1}{2}}, \text{ for } x \in D,$$

where $D \subseteq \mathbb{R}$ is the greatest possible domain of f .

- (a) Find the roots of $f(x) = 0$. (2)
- (b) Hence specify the set D . (2)
- (c) Find the coordinates of the local maximum on the graph $y = f(x)$. (2)
- (d) Solve the equation $f(x) = 3$. (3)
- (e) Sketch the graph of $|y| = f(x)$, for $x \in D$. (3)
- (f) Find the area of the region completely enclosed by the graph of $|y| = f(x)$. (3)

(Total 15 marks)